

Condensate Nuclei and Magnetic Polarity Reversals in the Sun and Solar-type Stars

Babur M. Mirza

Department of Mathematics,
Quaid-i-Azam University, 45320, Islamabad. PK.

May 29, 2015

Abstract

Magnetic field generation in the Sun and solar-type stars is modelled here based on the formation of magnetically polarized condensates (B. M. Mirza, *Mod. Phys. Lett. B* **28** (2014) 1450148) under the high density and pressure conditions. The correct orders of magnitude for the time period and the energy loss in a solar cycle are deduced, as well as the enhancement in the energy emission observed during solar cycles. It is shown that this feedback magnetic field along with differential rotation is sufficient to generate the toroidal magnetic field in the solar exterior. The model is useful in determining magnetic polarity reversals, energy generation, and reversal times, in solar-type stars.

1 Introduction

Magnetic field in normal stars like the Sun plays an important role almost all stellar activity. Magnetic field governs many processes on the Sun including the sunspot generation, particle acceleration effects causing solar prominences and solar wind, and numerous other aspects of the solar corona, the chromosphere and the photosphere [2]. In fact the Sun possesses a large-scale poloidal field that is particularly prominent in the polar regions and has a dipole symmetry. This magnetic field reverses polarity in a cycle of about 11 years. In general this cyclic activity is observed in most Sun-like stars with periods around 10 – 11 years [3].

On the solar surface, the deformation of toroidal field lines due to the differential rotation of the Sun accounts for the sunspot cyclic behavior associated with the solar cycle, particularly the pattern of sunspots around the equator (Spörer's law). There are various processes by which this is explained, including the formation and motion of magnetically active zones called the convection cells ([4-6], also for further references see Ref. [7]).

However, the continuous generation of the magnetic field requires a feedback mechanism. Observationally, this field is produced in the stellar core. In

fact almost all stellar energy, including the high poloidal magnetic field (about $10G - 10^3G$) energy of these stars, is generated in the stellar nucleus. Also, helioseismic measurements for instance, indicate extreme pressure and density conditions in the solar core. These conditions are sufficient to generate mass-energy conversion in the solar interior [2, 8].

Here, it is shown that condensate formation is an efficient mechanism for magnetic polarity reversals and energy generation in the Sun and other solar-type stars. The model is based on the elementary theory of magnetically polarized condensates.

The general plan of the work is as follows. Starting from the condensate energy equation (detailed derivation can be found in Ref. [1]), we first calculate (Section 2) the pressure required to counter-balance the gravitational pressure at the condensate surface. This implies the equilibrium condition under which a condensate can exist in the stellar core. This in turn yields a harmonic constraint on the condensate radius, whose amplitude and frequency are determined using quantum energy, and relativistic velocity conditions. We then calculate (in Section 3) the energy loss in the process. It is seen that the energy loss is maximum at each solar cycle periodically. The question of how this condensate energy is converted into the magnetic energy of the star is then discussed in Section 4. Here the electron-ion magnetic polarity is used to assign a micro magnetic field via the molecular magnetic field coefficient to the particles forming the condensate. It gives absolute magnetization as a measure for the magnetic field strength of the star. Based on this model, and incorporating the mass-energy conversion correction, correct order of magnitude for magnetic field reversal times (roughly equal to the 10 – 11 year solar cycle) is obtained. Furthermore, the magnetic field strength at the time of field reversals, shows a marked increase in magnitude, as observed during solar cycles. Section 5 gives a summary of the main results.

2 Condensate Pressure and the Equilibrium Condition

In normal main sequence stars, and generally for gravitationally bound systems, a stable equilibrium configuration exists when internal thermal pressure is counterbalanced by the gravitational pressure. This thermal pressure is primarily due to the mean kinetic energy of the ensemble under the ideal gas assumption. Inside the stellar core high density conditions force particles to coalesce, especially in the limiting condition for matter close to the center of the star.

Thermodynamically the pressure set up at any point in the star is given by the relation:

$$p = - \left(\frac{\partial \epsilon}{\partial V} \right)_S, \quad (1)$$

where S is the entropy, and ϵ is the macroscopic energy of the system. For a

condensate this energy can be calculated using the expression :

$$\epsilon = \frac{2\pi\hbar}{\bar{\lambda}} \bar{v} \tan \left[2\pi \left(\frac{\Delta x - \bar{v} \Delta t}{\bar{\lambda}} \right) \right]. \quad (2)$$

(see Ref. [1] for the derivation) where \bar{v} is the average particle speed and $\bar{\lambda}$ is the average particles wavelength for the system. Let the system has volume V containing N particles, then each linear dimension of this volume can be given as $\Delta x \approx (2^3 V/N)^{1/3}$. Thus the thermal (total) energy of the condensate is given by

$$\epsilon = h\nu \tan \left[\frac{2\pi}{\bar{\lambda}} \left(\frac{2^3 V}{N} \right)^{1/3} - 2\pi\bar{\nu}t \right]. \quad (3)$$

In equation (3) we have chosen $t_0 = 0$, thus we have substituted $\Delta t = t$. Also $\bar{\nu} = \bar{v}/\bar{\lambda}$ is the (average) particle related frequency. Using equations (1) and (3) we find that the pressure for the condensate is given by:

$$p = -\frac{\eta}{V^{2/3}} \sec^2 \left[\frac{2\pi}{\bar{\lambda}} \left(\frac{2^3 V}{N} \right)^{1/3} - 2\pi\bar{\nu}t \right], \quad (4)$$

where $\eta = 4\pi\hbar\bar{v}/3\bar{\lambda}^2 N^{1/3}$. This is the condensate pressure. We notice that it depends on the volume and number density of the particles in the condensate, and it gains a particularly large value when the condensate radius becomes small, such as due to gravitational contraction.

On the other hand, for a spherically symmetric mass distribution the gravitational pressure inside the star, at a distance R from the center is

$$p_g = \frac{3}{8\pi} \frac{GM^2}{R^4}, \quad (5)$$

where $M = M(R)$ is the total mass contained inside the sphere of radius R .

Now at each point of the condensate, at a distance R from the center, the outward condensate pressure (4) must equal the inward gravitational pressure p_g given by equation (5). Thus the equilibrium condition $p = -p_g$ for the condensate, say at the condensate surface R , imply that:

$$\frac{3GM^2}{8\pi\eta} \cos^2 \left[2\pi \left(\frac{\Delta x}{\bar{\lambda}} - 2\pi\bar{\nu}t \right) \right] = R^4 V^{-2/3}. \quad (6)$$

Since under the conditions the condensate has a spherical symmetry, therefore we take for its volume $V = 4\pi R^3/3$. This gives $R^4 V^{-2/3} = 4\pi R^2/3$ and equation (6) gives the condensate radius:

$$R(t) = \sqrt{\frac{9G\tilde{M}^2}{32\pi^2\eta}} \cos \left[2\pi \left(\frac{\Delta x}{\bar{\lambda}} - 2\pi\bar{\nu}t \right) \right]. \quad (7)$$

Equation (7) shows that the condensate radius, hence its volume, do not remain constant but periodically oscillate from maximum radius $R_{\max} = \sqrt{9GM^2/32\pi^2\eta}$

to zero. Also note that the condensate mass \tilde{M} here is small compared to the mass of the solar nuclei.

We next calculate the frequency and time period of this oscillation for a Sun-like star. First note that the periodicity of these oscillations depends on the average particle frequency $\bar{\nu}$. In general this frequency is given by the quantum condition $E = h\nu$. In a degenerate gas each electron/ion is trapped in a fixed region of space, say a box of length a , with energy

$$E = \frac{h^2}{8ma^2}. \quad (8)$$

Since it is the gravitational pressure of the surrounding gas that confines the electron within this region, therefore its kinetic energy is given by

$$\frac{mv^2}{2} = \frac{GMm}{a}. \quad (9)$$

Notice that as the stellar core contracts the particle speed v can be exceedingly large. In the limiting case this velocity has an upper bound given by the speed of light c . Thus combining equation (8) and (9), and using the quantum energy law $E = h\nu$, the particle related frequency in the condensate is given by $hc^2/16GMm$. This implies a fundamental mode of oscillations with frequency:

$$\Omega_C = \frac{h}{8m} \left(\frac{c^2}{2GM} \right)^2, \quad (10)$$

for the star, which depends only on the stellar mass M and the mass of the condensate particles. Putting in the for the parameters $M_\odot \approx 2 \times 10^{30} kg$, $G = 6.673 \times 10^{-11} Nm/kg^2$, $m = 9.109 \times 10^{-31} kg$, $c = 3 \times 10^8 m/sec$, and $h = 6.626 \times 10^{-34} J sec$, we obtain for the solar condensate frequency:

$$\Omega_{C\odot} \approx 0.1 \times 10^{-10} cycles/sec. \quad (11)$$

Correspondingly the time to complete one such cycle is

$$\tau_{C\odot} \approx 10^{11} sec. \quad (12)$$

This value differs by about 340 times from the solar cycle period of $11 yrs \approx 0.31536 \times 10^9 sec$. This difference in the values is due to mass-energy conversion in the stellar interiors. The relativistic correction to the time $\tau_{C\odot}$ (time-dilation) is of the order $10^{-2} sec$ for Sun. The calculated time of a solar cycle thus lies in the observed range of magnitudes.

3 Energy Loss and the Role of Thermal Pressure

An important aspect of the solar cycle is the enhanced energy generation during the process. In this respect condensate squeezing by the gravitational pressure

acts as an effective mechanism for energy extraction from gravitationally bound material systems. This process occurs at long time scales, as indicated by the formula (10) for the Sun-like stars.

Using the energy formula (2), we find that the energy loss $-d\epsilon/dt$ during a solar cycle is given by

$$-\frac{d\epsilon}{dt} = 2\pi\nu E_0 \sec^2 \left[\frac{2\pi \Delta x}{\lambda} - 2\pi\nu t \right], \quad (13)$$

where $E_0 = h\nu$ is the ground state (or minimum) energy of the bound quantized system. This energy loss periodically reaches a maximum after a time:

$$\tau_C = \frac{16G^2 M^2 m}{hc^4}. \quad (14)$$

The plot in Figure1 shows that the energy loss has an almost saturated value except at the states of cyclic periodicity, where it is particularly high.

Finally, we estimate the temperature attained in the stellar nuclei in this process. This can be derived by using the fact that the thermal equilibrium is maintained by the counterbalancing of the thermal and gravitational pressure (given by equation (5)), where as the thermal pressure is given by

$$p_T = nk_B T, \quad (15)$$

where $n = 3M/4\pi R^3 m_M$. Thus the temperature supplied to the condensate is:

$$T = \frac{Gm_H}{2k_B} \frac{M(R)}{R} \quad (16)$$

This shows that as the stellar nuclei contracts $R \rightarrow 0$, the temperature becomes exceedingly large. The cyclic energy loss results in heat generation in the stellar core, which in turn causes the general heating and enhanced radiation emission in the stellar exterior during the solar cycle..

4 Magnetic Polarity Reversals

The energy generated in a solar cycle strongly effects the solar magnetic field, in particular, it exhibits solar magnetic polarity reversals and an enhancement in the solar magnetic intensity.

We note that the condensate must have a resultant magnetic polarity since it is formed of particles possessing some net magnetic moment. This magnetic polarity can therefore be calculated from the molecular field coefficient and magnetic susceptibility of the system.

In general, the total energy ϵ and spontaneous magnetization I of a magnetized state of matter is related by

$$\epsilon = \epsilon_1 - wI^2/2, \quad (17)$$

where w is the molecular field coefficient, and ϵ_1 is the reference magnetic energy before magnetization. Also the magnetic susceptibility of a material is given by $\chi = dI/dB$, where B is the magnetic field intensity. From these relations, and the constant $\epsilon_1 = 0$, absolute magnetization can be expressed as

$$|I| = \sigma \tan^{1/2} \left[\frac{2\pi}{\lambda} \left(\frac{2^3 V}{N} \right)^{1/3} - 2\pi\nu t \right], \quad (18)$$

where σ denotes $(2E_0/w)^{1/2}$, and where use has been made of equation (3). Generally χ is not a function of H , so that the magnetic intensity is related to magnetization by the equation $\chi B = \pm I_0 \pm I$, where the \pm sign is due to the absolute value of I appearing in equation (18). Thus we have for the magnitude of the magnetic field

$$|B| = \frac{\sigma}{\chi} \tan^{1/2} \left[\frac{2\pi}{\lambda} \left(\frac{2^3 V}{N} \right)^{1/3} - 2\pi\nu t \right] + B_0 \quad (19)$$

where $B_0 = \pm I_0/\chi$. Since the magnetic field becomes imaginary for some values of t , to obtain the real values of the magnetic field B we multiply it with its complex conjugate, and then take the square-root. This is essentially a dipolar field, since magnetization I must have an orientation in space. Also, since this field is generated in the stellar interior without a rotational mechanism, it must be identified with the primary poloidal magnetic field of the star. A plot of this magnetic field B is given in Fig.2, which shows a polarity reversal and a particularly enhanced magnetic field intensity near after each cycle.

5 Conclusions

The above study leads to the following results:

- (1) Under the extreme density and pressure, along with a high temperature, maintained inside a Sun-like star, matter in normal stellar interiors transforms into a condensate-like coherent state.
- (2) This condensate however is not stabilized against the gravitational pressure, which causes it to decay in size.
- (3) The gravitational squeeze-in of the condensate act as an active mechanism of converting stellar matter into energy in the form of magnetic field, and this occurs cyclically, resulting in the magnetic pole reversals and heightened magnetization of the Sun.
- (4) The condensate nucleus has a magnetic polarity and causes the generation of the feedback poloidal magnetic field of the star. This poloidal field along with the differential rotation of the star, causes the toroidal field generation, and the observed cyclicity in the sunspot behavior.

Since condensate formation under high density conditions must be a rather common feature in all gravitationally bound dense systems, it is of interest to study its role in such systems as well, particularly in compact stars and in the problem of gravitational stability (see Ref.[9]).

References

- [1] Mirza, B. M. *Mod. Phys. Lett. B* **28** (2014) 1450148.
- [2] Stix, M. (2002), *The Sun, An Introduction*. 2nd ed. Springer, Berlin. Ch. 8, also Ch.3 and 6.
- [3] Baliunas, S. L., et. al. (1995) *ApJ* **438**, 269-287.
- [4] Miesch, M. S. (2008) *Journal of Physics: Conference Series*.**18**, 012031.
- [5] Parker, E. N. (2009) in *The Origin and Dynamics of Solar Magnetism*. Springer, New York. pp. 15-24.
- [6] Tobias, S. and Weiss, N. (2007) in *Mathematical Aspects of Natural Dynamos*, ed. E. Dormy, and A. M. Soward, CRC Press, Taylor and Frances Publishers, New York. pp.280-311.
- [7] Thompson, M. J. et.al. (2011) *The Origin and Dynamics of Solar Magnetism*. Springer, New York.
- [8] Gough, D. (2012) *arXiv:1210.0820v1 [astro-ph.SR]*.
- [9] Mirza, B. M. (to appear).

Figure Captions:

Figure 1: Energy loss due to condensate formation-decay oscillation per solar cycle ($\approx 0.31536 \times 10^9$ sec), for $\Delta x = 4\bar{\lambda}$. The net energy loss is divided here by $2\pi\nu E_0$ units, in equation (13).

Figure 2: Magnetic polarity reversals in a solar-type star ($M = M_\odot$), with periodicity determined from equation (14) and relativistic correction of factor 10^{-2} sec. Here $B_0 = 0$, $\Delta x = \bar{\lambda}$, and $\sigma = -\chi$.

6

This figure "Figure_1.GIF" is available in "GIF" format from:

<http://arxiv.org/ps/1505.07642v1>

This figure "Figure_2.GIF" is available in "GIF" format from:

<http://arxiv.org/ps/1505.07642v1>